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Modal Analysis of a Discrete System in the Form of a Rocket Launcher Installed on a Motor Vehicle

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Abstract. A discrete model of an unguided rocket missile launcher installed on a motor vehicle was developed on the basis of a real assembly. The model is simplified to a vertical plane; it has four degrees of freedom and is adapted for a modal analysis. A mathematical model was derived using the variational method on the basis of the assumed physical model. Analytical dependencies in the form of adjoint second order equations taking ordinary derivatives describe the dynamics of the considered system. The equations describing the autonomous motion of the system provided a definition of the generalized eigenvalue problem and the age equation forming the basis of the characteristic values. An analytical form of particular integrals was assumed for equations describing proper vibrations and the amplitude distribution coefficients were calculated. Four forms of proper vibrations of the considered system were obtained on the basis of the determined eigenvectors.
Keywords: mechanics, rocket launcher, armoured combat vehicle, eigenvalues and eigenvectors

1. INTRODUCTION

The object of this study is a remotely controlled weapon module designated ZSMU-70 and shown in Fig. 1. The ZSMU-70 is a mobile rocket missile system, comprising an armoured motor vehicle, which forms a platform for the WW-4 rocket launcher intended for NLPR-70 unguided rocket missiles.

The ZMSU-70 in this configuration can be deployed against light armoured land vehicles. The live firing range tests of this weapon module were recorded with a Phantom v9.1 high-speed slow-motion digital camera. The motion of the ZSMU-70 recorded during the live firing range tests positively demonstrated the feasibility of simplifying its 3D model to two dimensions. The system contemplated herein has symmetrical geometrical and inertial characteristics; hence the virtual model was defined on a vertical plane. This work presents a modal analysis of the contemplated discrete system.

The monograph [8] describes a carrier in the form of a relatively small motor vehicle operated as a launch platform for short-range surface-to-air homing missiles. The rocket missiles this paper concerns are unguided. In unguided rockets missiles, other properties exist that make all the processes and physical phenomena which occur during their launch and flight different from those already known. The theoretical contemplation herein concerns a specific weapon module, the ZSMU-70, and follows its live firing range tests. The monograph exclusively concerns a theoretical analysis whose results were not verified by experimental research. There was also no modal analysis of the developed discrete systems. Naturally, the experience gained by testing anti-aircraft weapon systems with homing missiles permitted an informed and universal formulation of the objectives, methodology and a plan of the contemplation of unguided rocket missile systems.

Fig. 1. Mobile rocket missile system

2. PHYSICAL MODEL

Given the objective of this work, the contemplated system was modelled as a system of three masses and four deformable elements [6, 10], as shown in Fig. 2.

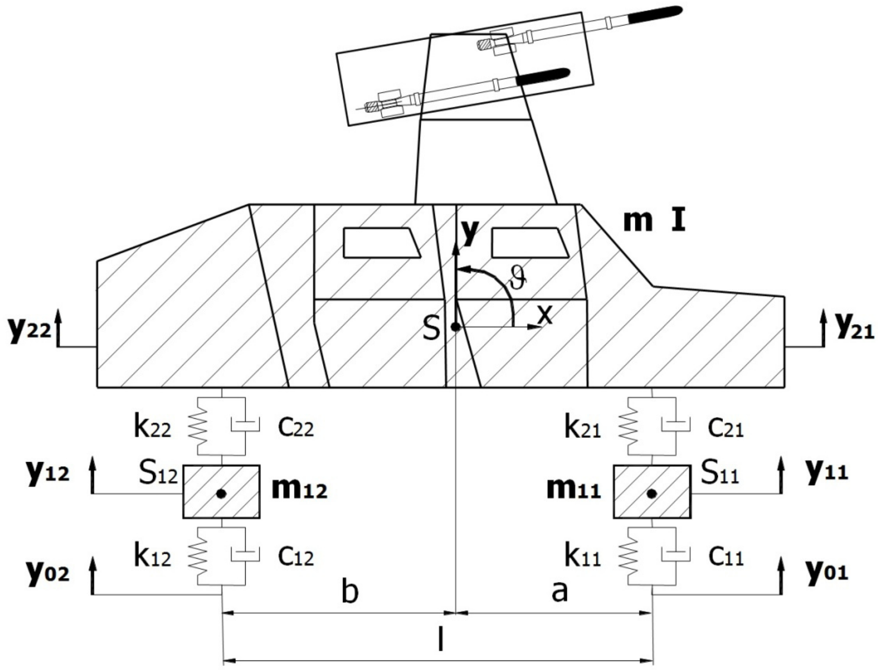


Fig. 2. Physical model of the mobile rocket missile system

The bodywork of the carrier motor vehicle is a perfectly rigid body. The axles and the wheels of the motor vehicle are reduced to two concentrated masses, m_{11} and m_{12}. The characteristics of radial flexibility of front and rear wheel tyres and the characteristics of the front and rear suspension are expressed as Kelvin–Voigt rheological models. Hence the characteristics of weightless deformable elements are represented with the following coefficients of rigidity and attenuation: k_{11} and c_{11}, being the characteristics of flexibility and dissipation of the front wheel tires; k_{12} and c_{12}, being the characteristics of flexibility and dissipation of the rear wheel tires; k_{21} and c_{21}, being the characteristics of flexibility and dissipation of the front axle suspension; and k_{22} and c_{22}, being the characteristics of flexibility and dissipation of the rear axle suspension. The carrier motor vehicle has a rocket missile launcher installed which comprises a turret and a system of launcher guides [3, 4, 5, 13].

All these objects are assumed to form perfectly rigid bodies. Each launcher guide houses a rocket missile, also modelled as a perfectly rigid body. The mass m and the moment of inertia I are reduced parameters of a system comprising the motor vehicle bodywork plus the rocket missile launcher with its ammunition. The assumption is that these object form a single perfectly rigid body.

The system may be subject to an external input from the road surface. The external input is kinematic and defined with the path of vertical displacement at the wheel contact with the road surface, y_{01} and y_{02}. This contemplation assumes the external input as a predetermined model of input signals.

The number of the degrees of freedom from the formulated system structure is four (4).

3. MATHEMATICAL MODEL

The physical model explained above was converted into a mathematical model [9, 11] with a strain energy method of Lagrange second kind equations. The motion of the developed model was determined with four independent coordinates $y, \vartheta, y_{11}, y_{12}$ and four static displacement coordinates $y_{st}, \vartheta_{st}, y_{11st}, y_{12st}$.

Equations of the system motion in the simplified form (1):

$$
\begin{aligned}
& m\ddot{y} + c_{21}\dot{\lambda}_{21} + c_{22}\dot{\lambda}_{22} + k_{21}\lambda_{21} + k_{22}\lambda_{22} = -mg \\
& I\ddot{\vartheta} + c_{21}a\dot{\lambda}_{21} - c_{22}b\dot{\lambda}_{22} + k_{21}a\lambda_{21} - k_{22}b\lambda_{22} = 0 \\
& m_{11}\ddot{y}_{11} + c_{11}\dot{\lambda}_{11} - c_{21}\dot{\lambda}_{21} + k_{11}\lambda_{11} - k_{21}\lambda_{21} = -m_{11}g \\
& m_{12}\ddot{y}_{12} + c_{12}\dot{\lambda}_{12} - c_{22}\dot{\lambda}_{22} + k_{12}\lambda_{12} - k_{22}\lambda_{22} = -m_{12}g
\end{aligned} \tag{1}
$$

with:

$$
\begin{aligned}
& \lambda_{11} = y_{11} + y_{11st} - y_{01} \\
& \lambda_{12} = y_{12} + y_{12st} - y_{02} \\
& \lambda_{21} = y + y_{st} + a(\vartheta + \vartheta_{st}) - y_{11} - y_{11st} \\
& \lambda_{22} = y + y_{st} - b(\vartheta + \vartheta_{st}) - y_{12} - y_{12st} \\
& \dot{\lambda}_{11} = \dot{y}_{11} - \dot{y}_{01} \\
& \dot{\lambda}_{12} = \dot{y}_{12} - \dot{y}_{02} \\
& \dot{\lambda}_{21} = \dot{y} + a\dot{\vartheta} - \dot{y}_{11} \\
& \dot{\lambda}_{22} = \dot{y} - b\dot{\vartheta} - \dot{y}_{12}
\end{aligned}
$$

Equations of the system equilibrium (2):

$$
\begin{aligned}
&k_{21}\left(y_{st}+a\vartheta_{st}-y_{11st}\right)+k_{22}\left(y_{st}-b\vartheta_{st}-y_{12st}\right)+mg=0\\
&k_{21}a\left(y_{st}+a\vartheta_{st}-y_{11st}\right)-k_{22}b\left(y_{st}-b\vartheta_{st}-y_{12st}\right)=0\\
&k_{11}y_{11st}-k_{21}\left(y_{st}+a\vartheta_{st}-y_{11st}\right)+m_{11}g=0\\
&k_{12}y_{12st}-k_{22}\left(y_{st}-b\vartheta_{st}-y_{12st}\right)+m_{12}g=0
\end{aligned}
\tag{2}
$$

4. SYSTEM EIGENVALUES

The modal analysis is carried out by reducing the mathematical model of the contemplated weapon system to a conservative autonomous system [12, 15]. The result is a system of four linear homogeneous ordinary differential equations:

$$
\begin{aligned}
&m\ddot{y}+k_{21}\left(y+a\vartheta-y_{11}\right)+k_{22}\left(y-b\vartheta-y_{12}\right)=0\\
&m\ddot{\vartheta}+k_{21}a\left(y+a\vartheta-y_{11}\right)-k_{22}b\left(y-b\vartheta-y_{12}\right)=0\\
&m_{11}\ddot{y}_{11}+k_{11}y_{11}-k_{21}\left(y+a\vartheta-y_{11}\right)=0\\
&m_{12}\ddot{y}_{12}+k_{12}y_{12}-k_{22}\left(y-b\vartheta-y_{12}\right)=0
\end{aligned}
\tag{3}
$$

Model parameters:

$m=1780\,[\mathrm{kg}]$	$I=2620\left[\mathrm{kgm}^2\right]$	
$m_{11}=113\,[\mathrm{kg}]$	$m_{12}=157\,[\mathrm{kg}]$	
$k_{11}=350000\left[\frac{\mathrm{N}}{\mathrm{m}}\right]$	$k_{12}=400000\left[\frac{\mathrm{N}}{\mathrm{m}}\right]$	
$k_{21}=75000\left[\frac{\mathrm{N}}{\mathrm{m}}\right]$	$k_{22}=65000\left[\frac{\mathrm{N}}{\mathrm{m}}\right]$	
$a=1{,}14\,[\mathrm{m}]$	$b=1{,}28\,[\mathrm{m}]$	$l=2{,}42\,[\mathrm{m}]$

The generalized eigenvalue problem is solved, and an age equation is derived with an eigenvalue determinant.

$$
b_4\omega_0^8+b_3\omega_0^6+b_2\omega_0^4+b_1\omega_0^2+b_0=0 \tag{4}
$$

with:

$$
b_0=k_{11}k_{12}k_{21}k_{22}(a+b)^2
$$

$$
\begin{aligned}
b_1=&-\left[\left(k_{12}+k_{22}\right)k_{11}k_{21}a^2+\left(k_{11}+k_{21}\right)k_{12}k_{22}b^2\right]m-\left[\left(k_{21}+k_{22}\right)k_{11}k_{12}+\right.\\
&\left.\left(k_{11}+k_{12}\right)k_{21}k_{22}\right]I-k_{12}k_{21}k_{22}(a+b)^2m_{11}-k_{11}k_{21}k_{22}(a+b)^2m_{12}
\end{aligned}
$$

$$b_2 = \left[\left(k_{12}b^2 + k_{21}a^2\right)k_{22} + k_{12}k_{21}a^2\right]mm_{11} + \left[\left(k_{21}a^2 + k_{22}b^2\right)k_{11} + k_{21}k_{22}b^2\right]mm_{12} +$$
$$+ \left[\left(k_{12} + k_{21}\right)k_{22} + k_{12}k_{21}\right]I\,m_{11} + \left[\left(k_{21} + k_{22}\right)k_{11} + k_{21}k_{22}\right]I\,m_{12} +$$
$$+ k_{21}k_{22}(a+b)^2 m_{11}m_{12} + \left(k_{11} + k_{21}\right)\left(k_{12} + k_{22}\right)mI$$
$$b_3 = -\left(k_{21}a^2 + k_{22}b^2\right)mm_{11}m_{12} - \left(k_{21} + k_{22}\right)Im_{11}m_{12} - \left(k_{12} + k_{22}\right)mIm_{11} +$$
$$- \left(k_{11} + k_{21}\right)mIm_{12}$$
$$b_4 = mIm_{11}m_{12}$$

The age equation (4) is applied to determine the eigenvalues of the system. The eigenvalues are dynamic characteristics of the mobile rocket missile system. The distribution of the dynamic characteristics depends on the parameter values of the system. The polynomial radicals were determined by applying an 'fsolve' inclusive algorithm in Scilab.

$$\omega_{01} = 8.0633731 < \omega_{02} = 8.1365435 < \omega_{03} = 54.523538 < \omega_{04} = 61.443545 \left[\frac{\text{rad}}{\text{s}}\right] \tag{5}$$

5. SYSTEM EIGENVECTORS

The eigenvectors of the system are determined at the known eigenfrequencies by formulating particular integrals and substituting them in differential equations (3) [1, 2, 14]. Following these transformations, each eigenfrequency has three algebraic equations which are solved to determine the distribution coefficient values for the proper vibration amplitudes.

The particular integrals of the differential equations (3) formulated for the eigenfrequency ω_{0i} $-$ *gdzie*: $i = 1, 2, 3, 4$ are:

$$\begin{aligned} y_i &= A_{1i}\sin\left(\omega_{0i}t + \alpha_i\right) \\ \vartheta_i &= A_{2i}\sin\left(\omega_{0i}t + \alpha_i\right) \\ y_{11}^i &= A_{3i}\sin\left(\omega_{0i}t + \alpha_i\right) \\ y_{12}^i &= A_{4i}\sin\left(\omega_{0i}t + \alpha_i\right) \end{aligned} \tag{6}$$

The algebraic equations used to determine the distribution coefficient values for the proper vibration amplitudes:

$$\begin{aligned} &\left(k_{21} + k_{22} - m\omega_{0i}^2\right)\mu_{1i} + \left(k_{21}a - k_{22}b\right)\mu_{2i} - k_{21}\mu_{3i} - k_{22}\mu_{4i} = 0 \\ &- k_{21}\mu_{1i} - k_{21}a\mu_{2i} + \left(k_{11} + k_{21} - m_{11}\omega_{0i}^2\right)\mu_{31} = 0 \\ &- k_{22}\mu_{1i} + k_{22}b\mu_{2i} + \left(k_{12} + k_{22} - m_{12}\omega_{0i}^2\right)\mu_{41} = 0 \end{aligned} \tag{7}$$

with:

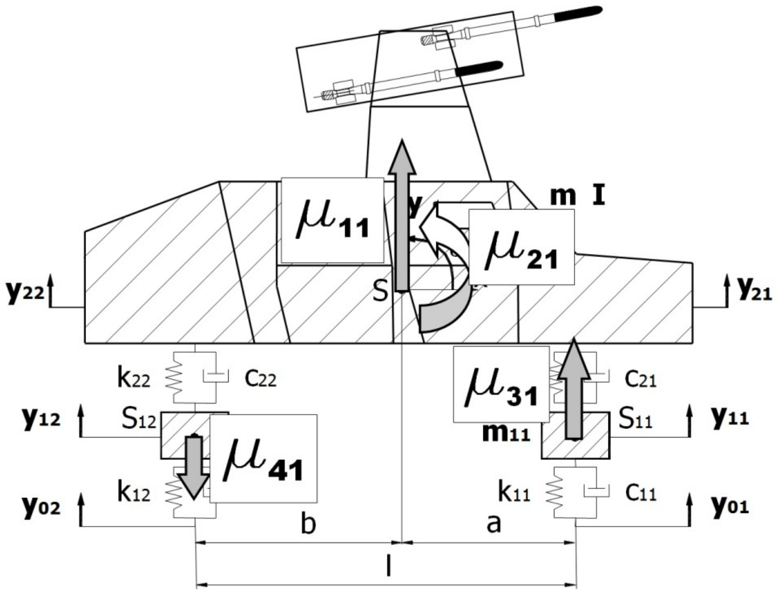
$$\mu_{1i} = \frac{A_{1i}}{A_{1i}} = 1 \qquad \mu_{2i} = \frac{A_{2i}}{A_{1i}} \qquad \mu_{3i} = \frac{A_{3i}}{A_{1i}} \qquad \mu_{4i} = \frac{A_{4i}}{A_{1i}}$$

The algebraic equations (7) are applied to determine the eigenvectors of the system. The eigenvectors are dynamic characteristics of the mobile rocket missile system. The distribution of the dynamic characteristics depends on the parameter values and structure of the system.

Proper vibration first form

$$\omega_{01} = 8.0633731 \quad \left[\frac{\text{rad}}{\text{s}}\right]$$

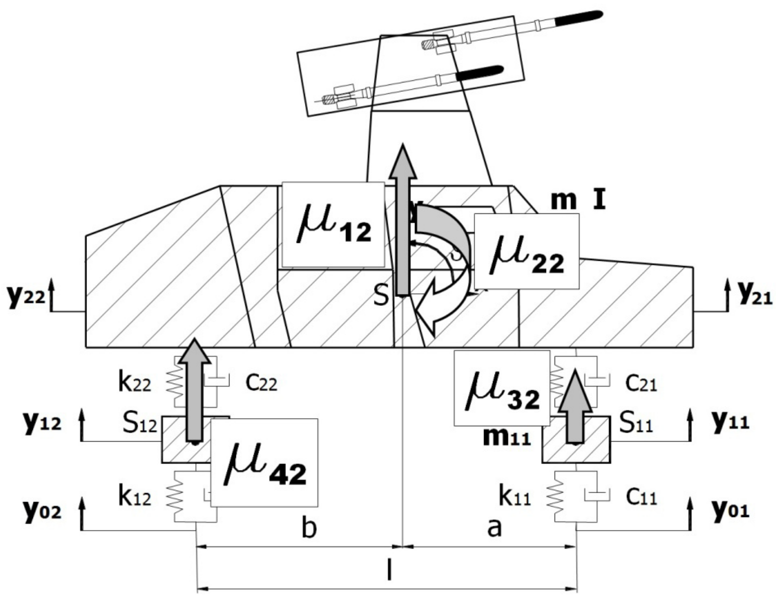
$$\mu_{11} = 1 \qquad \mu_{21} = 1{,}2988344 \qquad \mu_{31} = 0{,}4454664 \qquad \mu_{41} = -0{,}0946873$$



Proper vibration second form

$$\omega_{02} = 8.1365435 \quad \left[\frac{\text{rad}}{\text{s}}\right]$$

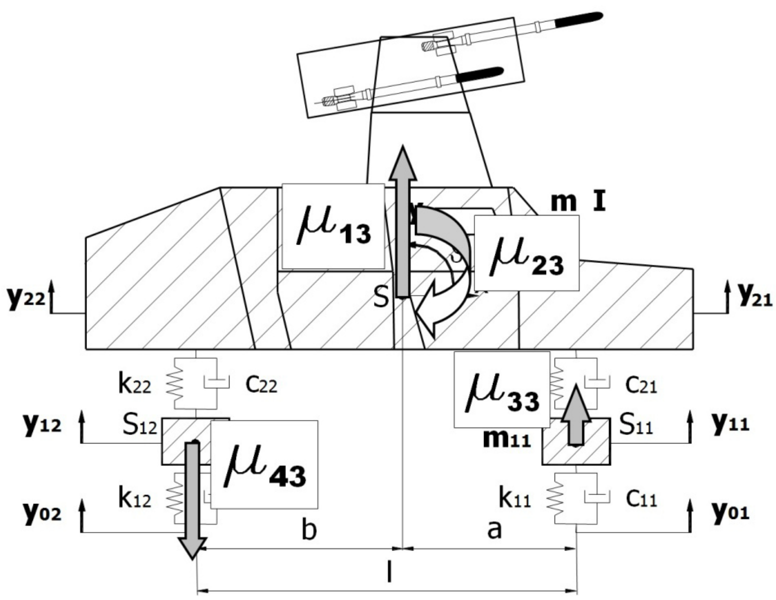
$$\mu_{12} = 1 \qquad \mu_{22} = -0{,}5231057 \qquad \mu_{32} = 0{,}0725104 \qquad \mu_{42} = 0{,}2387174$$



Proper vibration third form

$$\omega_{03} = 54.523538 \quad \left[\frac{\text{rad}}{\text{s}}\right]$$

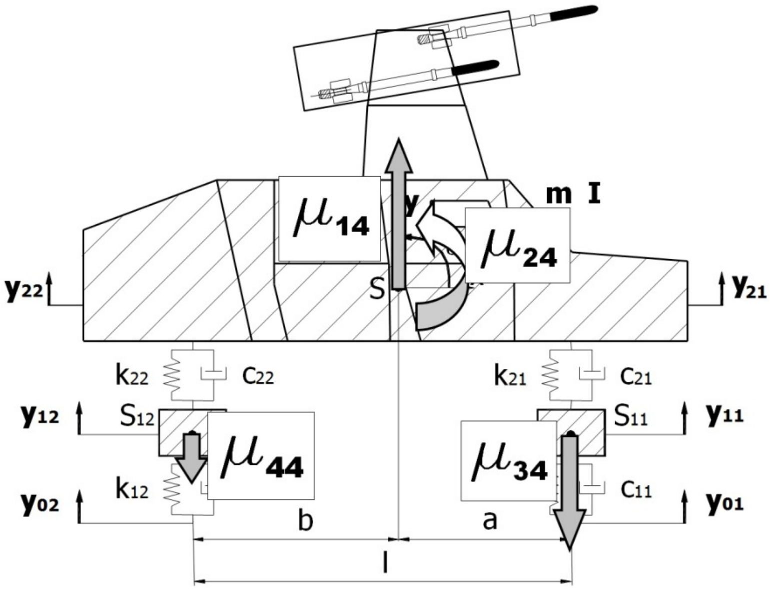
$\mu_{13} = 1 \qquad \mu_{23} = -0,8695898 \qquad \mu_{33} = 0,0072983 \qquad \mu_{43} = -79,294773$



Proper vibration fourth form

$$\omega_{04} = 61.443545 \quad \left[\frac{\text{rad}}{\text{s}}\right]$$

$\mu_{14} = 1 \quad \mu_{24} = 0{,}7742938 \quad \mu_{34} = -87{,}706333 \quad \mu_{44} = -0{,}0045313$



6. CONCLUSIONS

The dynamic characteristics of the mobile rocket missile system were determined as eigenvalues and eigenvectors. The modal analysis of the discrete system permits a preliminary estimation of the dynamics of the ZSMU-70. The authors hereof are unaware of any work with similar considerations of analogous discrete systems.

The rocket missile launcher has a direct effect on the inputs of the rocket missiles during the launch. When aggregated, the mass and the moment of inertia of the carrier vehicle bodywork plus the rocket missile launcher have a significant effect on the dynamics of the entire system. The derived relations facilitate the modal analysis of the system with various rocket launcher configurations, e.g. with the front of a rocket missile leaving the launcher guide.

The formulated model permits consideration of the motions of the model in the vertical plane. Based on the analysis of the images captured by the digital camera, the configuration present in the system's live firing range tests suggests that the motion in the transverse plane is negligible. The results of the analysis of the model in the vertical plane facilitate investigations into a 3D model. This eliminates the potential for errors that might result in far more complex systems.

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Analiza modalna układu dyskretnego w postaci wyrzutni rakietowej zabudowanej na pojeździe samochodowym

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Streszczenie. W pracy na podstawie rzeczywistego układu opracowany został dyskretny model wyrzutni niesterowanych pocisków rakietowych zabudowanej na pojeździe samochodowym. Model uproszczony został do płaszczyzny pionowej, ma cztery stopnie swobody i przystosowany jest do przeprowadzenia analizy modalnej. Na podstawie przyjętego modelu fizycznego z wykorzystaniem metody wariacyjnej został wyprowadzony model matematyczny. Zależności analityczne w postaci sprzężonych równań różniczkowych drugiego rzędu o pochodnych zwyczajnych opisują dynamikę rozpatrywanego układu. Na podstawie równań opisujących autonomiczny ruch układu określono uogólnione zagadnienie własne i otrzymano równanie wiekowe, na podstawie którego wyznaczono wartości własne. Przyjęto postać analityczną całek szczególnych dla równań opisujących drgania własne i obliczono współczynniki rozkładu amplitud. Na podstawie wyznaczonych wektorów własnych otrzymano cztery postacie drgań własnych rozpatrywanego układu.

Słowa kluczowe: mechanika, wyrzutnia rakiet, pojazd samochodowy, wartości i wektory własne